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The use of porous screens as wave dampers in narrow wave tanks

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Abstract. Expressions are derived for the reflection of plane incident waves from the closed end of a narrow wave tank when a number of thin vertical porous screens are introduced to damp the waves. The results may have application to the design of wave tanks where a small amount of beach reflection is desirable.

1. Introduction

One of the major problems associated with experiments in narrow wave tanks is unwanted wave reflections. The usual way of avoiding this is to introduce a sloping beach made of say, horse-hair or similar material, in order to dissipate the incident wave energy. An alternative method has recently been proposed (Pawlowski, personal communication) which involves using a number of thin perforated vertical metal screens suitably positioned next to a vertical wall. In the present work we examine this idea theoretically on the assumption that the dissipative properties of a single screen are known from experiment. Thus Tuck [1, p. 118 et seq.] discusses the application of a Darcy law of flow across such a screen and suggests that in the special case of sinusoidal oscillations in time the velocity across the screen should be related to the pressure drop by a complex-valued frequency-dependent parameter which accounts for both viscous and inertial effects. Such an assumption will be made here, and a matrix method described which enables the reflection from any number of screens in the presence of the wall, to be determined.

The problem of inserting screens in a *closed* rectangular tank of fluid is also considered. In the absence of such screens the fluid will oscillate with well-defined frequencies and with negligible damping. In some circumstances, for example, in the transportation of dangerous fluids, it may be desirable to introduce such screens in order to damp out unwanted oscillations. The method described here enables equations describing the complex frequencies to be determined for n identical equally-spaced screens.

2. Formulation

We choose two-dimensional Cartesian co-ordinates with y vertically upwards and x to the right. The bottom of the tank is $y = 0$, and the undisturbed free surface is $y = h$. The vertical wall occupies $x = 0$, $0 \leq y \leq h$, and the m^{th} screen $x = a_m$, $0 \leq y \leq h$, $m = 1, 2, \ldots, n$. On the basis of linearised water-wave theory there exists a velocity potential $\Phi(x, y, t) = \text{Re}[\phi(x, y)\, e^{-i\omega t}]$ where $\phi(x, y)$ satisfies

$$\nabla^2\phi = 0 \quad \text{in the fluid,} \tag{2.1}$$

$$K\phi - \phi_y = 0 \quad \text{on } y = h\,, \quad K = \omega^2/g\,, \tag{2.2}$$

$$\phi_y = 0 \quad \text{on } y = 0\,, \tag{2.3}$$

$$\phi_x = 0 \quad \text{on } x = 0\,. \tag{2.4}$$

In our first problem we shall assume that a plane wave of frequency ω and amplitude A is incident from $x = +\infty$ and is partially reflected back to positive infinity after undergoing multiple reflections at the screens and the wall. Thus we assume

$$\phi(x, y) \sim \frac{gA}{\omega}\,[\mathrm{e}^{-ik_0x} + R_n\,\mathrm{e}^{ik_0x}]\cosh k_0(h - y)\,, \quad x \to +\infty\,, \tag{2.5}$$

where k_0 is the positive real root of

$$\omega^2/g = K = k_0 \tanh k_0h\,. \tag{2.6}$$

We seek to minimise R_n by judicious choice of the position of the screens.

It is convenient to separate the problem into regions. Thus the m^{th} region is $a_{m-1} \leqslant x \leqslant a_m$, $0 \leqslant y \leqslant h$, $m = 2, \ldots, n$, having the $(m-1)^{\text{th}}$ screen as the left boundary, the m^{th} screen as the right. We choose $a_0 = 0$ so that the first region lies between the wall at $x = 0$ and the first screen.

We assume that within each region m there exists a potential $\phi_m(x, y)$ satisfying conditions (2.1), (2.2) and (2.3). In addition on the m^{th} screen we require that the horizontal velocity of the fluid be continuous:

$$\frac{\partial\phi_m}{\partial x} = \frac{\partial\phi_{m+1}}{\partial x}\,, \quad x = a_m\,, \quad m = 1, 2, \ldots n\,, \tag{2.7}$$

whilst

$$\frac{\partial\phi_1}{\partial x} = 0 \quad \text{on } x = 0\,, \tag{2.8}$$

where $\phi_{n+1}(x, y)$ is the potential in $x > a_n$ having asymptotic behaviour for large x given by (2.5).

We also assume a Darcy law at each screen of the form

$$U_m = c_m[p_m] \tag{2.9}$$

where U_m is the horizontal velocity and

$$[p_m] = -\rho i\omega\{\phi_{m+1} - \phi_m\}\,, \quad x = a_m\,, \quad m = 1, 2, \ldots, n\,, \tag{2.10}$$

is the jump in dynamic pressure across the m^{th} screen. Tuck [1] has suggested that a good empirical approximation to the inclusion of both inertial and viscous permeabilities is given by assuming

$$c_m = \kappa_m[1 + 2\rho C_m \kappa_m(-\mathrm{i}\omega)]^{-1} \tag{2.11}$$

where κ_m measures viscous dissipation at the screen and C_m^{-1} is the net blockage coefficient for the screen (Tuck [1], eqn. (5.29)).

Combining (2.9) and (2.10) gives

$$\frac{\partial \phi_m}{\partial x} = -\mathrm{i}\omega\rho c_m\{\phi_{m+1} - \phi_m\}\,, \quad x = a_m\,, \quad m = 1, 2, \ldots, n\,, \tag{2.12}$$

where c_m is a frequency-dependent complex parameter, presumed known.

We can separate out the y-dependence by writing

$$\phi_m(x, y) = \frac{gA}{\omega}\, \psi_m(x) \cosh k_0 y \tag{2.13}$$

where (2.13) satisfies (2.2), (2.3). Then $\psi_m(x)$ must satisfy

$$\psi_m''(x) + k_0^2 \psi_m(x) = 0\,, \qquad x \geq 0\,, \tag{2.14}$$

$$\psi_m'(x) = 0\,, \qquad x = 0\,, \tag{2.15}$$

$$\left.\begin{aligned} &\psi_m'(x) = \psi_{m+1}'(x) \qquad &(2.16)\\ &\psi_{m+1}'(x) = \mathrm{i}\omega\rho c_m(\psi_{m+1} - \psi_m) \qquad &(2.17)\end{aligned}\right\} x = a_m\,, \quad m = 1, 2, \ldots, n\,.$$

We now let

$$\psi_{m+1}(x) = A_m\, \mathrm{e}^{\mathrm{i}k_0(x - a_m)} + B_m\, \mathrm{e}^{-\mathrm{i}k_0(x - a_m)}\,, \quad m = 0, 1, \ldots, n\,, \tag{2.18}$$

where choosing $A_0 = B_0$ ensures that (2.15) is satisfied, and choosing

$$A_n\, \mathrm{e}^{-\mathrm{i}k_0 a_m} = R_n\,, \quad B_n\, \mathrm{e}^{\mathrm{i}k_0 a_n} = 1 \tag{2.19}$$

ensures that (2.5) is satisfied for $x \geq a_n$. Application of (2.16) gives

$$A_m - B_m = A_{m-1}\, \mathrm{e}^{\mathrm{i}k_0 b_m} - B_{m-1}\, \mathrm{e}^{-\mathrm{i}k_0 b_m}\,, \quad m = 1, 2, \ldots, n\,, \tag{2.20}$$

where $b_m = a_m - a_{m-1}$, and (2.17) gives

$$-2\beta_m(A_m - B_m) = A_m + B_m - A_{m-1}\, \mathrm{e}^{\mathrm{i}k_0 b_m} - B_{m-1}\, \mathrm{e}^{-\mathrm{i}k_0 b_m}\,, \quad m = 1, 2, \ldots, n\,, \tag{2.21}$$

where

$$2\mu_m = k_0/\rho\omega c_m\,, \quad \text{assumed known.} \tag{2.22}$$

We can solve (2.20), (2.21) for A_m, B_m in terms of A_{m-1}, B_{m-1}. Thus

$$\begin{pmatrix} A_m \\ B_m \end{pmatrix} = T_m \begin{pmatrix} A_{m-1} \\ B_{m-1} \end{pmatrix}, \quad m = 1, 2, \ldots, n\,, \tag{2.23}$$

where

$$T_m = \begin{pmatrix} (1-\mu_m)\,e^{ik_0 b_m} & \mu_m\,e^{-ik_0 b_m} \\ -\mu_m\,e^{ik_0 b_m} & (1+\mu_m)\,e^{-ik_0 b_m} \end{pmatrix}. \tag{2.24}$$

It follows from (2.19) that

$$\begin{pmatrix} R_n\,e^{ik_0 a_n} \\ e^{-ik_0 a_n} \end{pmatrix} \equiv \begin{pmatrix} A_n \\ B_n \end{pmatrix} = T\begin{pmatrix} A_0 \\ B_0 \end{pmatrix} \equiv T\begin{pmatrix} 1 \\ 1 \end{pmatrix} A_0 \tag{2.25}$$

where

$$T = T_1 T_2 \cdots T_n \equiv \begin{pmatrix} \alpha_n & \beta_n \\ \gamma_n & \delta_n \end{pmatrix}, \quad \text{say.} \tag{2.26}$$

Thus

$$R_n\,e^{2ik_0 a_n} = (\alpha_n + \beta_n)/(\gamma_n + \delta_n)\,. \tag{2.27}$$

Equation (2.27) provides an explicit expression for the reflection coefficient R_n in terms of the elements of the matrix T formed from the product over $m = 1, \ldots, n$ of the matrices T_m given by (2.24). The result (2.27) is exact under the assumptions of the problem and incorporates the different spacings of the screens and the separate empirical permeability constants μ_m for each screen. Bearing in mind that in general each μ_m is a frequency-dependent complex quantity, the task of optimising the spacing and the number of screens in order to achieve a small value of $|R_n|$ over a range of incident wavelengths is a daunting prospect. In the case of a single screen, $n = 1$ and we have from (2.24), (2.25) with $m = 1$, $a_m = b_m = a$, say, and $\mu_1 = \mu$,

$$R_1\,e^{2ik_0 a} = \frac{(1-\mu)\,e^{ik_0 a} - \mu\,e^{-ik_0 a}}{(1+\mu)\,e^{-ik_0 a} - \mu\,e^{ik_0 a}} \tag{2.28}$$

$$= \frac{\cos k_0 a + (1-2\mu)\mathrm{i}\sin k_0 a}{\cos k_0 a - (1+2\mu)\mathrm{i}\sin k_0 a}\,. \tag{2.29}$$

Now $\mu = k_0/2\rho\omega c_1$ from (2.21) and $\mu = 0$ corresponds to $c_1 = \infty$ and, from (2.12), to continuity of the potentials across the screen.

It follows, as expected, from (2.28) that in this case $R_1 = 1$, total reflection occurring from the rigid wall at $x = 0$. Conversely if $\mu \to \infty$, $c_1 \to 0$ and from (2.12) the screen is impervious to the flow, whence $R_1 = e^{-2ik_0 a}$, the phase factor reflecting the fact that the wave is completely reflected by the screen at $x = a$.

In the absence of the back wall at $x = 0$, a transmitted wave would exist travelling undiminished towards $x = -\infty$. The reflection coefficient in this case is obtained by relaxing the condition $A_0 = B_0$ and instead choosing $A_0 = 0$ to ensure no wave is reflected from $x = 0$. The reflection coefficient in this case, for n screens, is now given from (2.24) with $A_0 = 0$. Thus eliminating B_0,

$$R_n^{\infty}\,e^{2ik_0 a_n} = \beta_n/\delta_n \tag{2.30}$$

where the superscript ∞ indicates the wall at $x = 0$ is no longer present. For $n = 1$ we obtain

$$R_1^\infty \, e^{2ik_0 a} = \mu/(1+\mu)\,, \tag{2.31}$$

$$|R_1^\infty| = |\mu|/|1+\mu|\,, \tag{2.32}$$

showing how the effect of the screen, through μ, affects the amplitude of the incident wave in an infinitely long tank. The combination of both screen *and* end wall provides the possibility of considerable reduction in $|R_1|$. Thus it follows from (2.29) that $R_1 = 0$ when

$$\cos k_0 a = (2\mu - 1)\mathrm{i} \sin k_0 a\,, \tag{2.33}$$

which, since $k_0 a$ is real, can only be achieved if

$$2\mu - 1 = -\mathrm{i}\lambda\,, \quad \lambda \text{ real}, \tag{2.34}$$

when the condition for $R_1 = 0$ becomes

$$\cot k_0 a = \lambda\,, \quad \text{when } \lambda = 2k_0 C_1 + \mathrm{i}(k_0/\kappa_1 \rho\omega - 1) \tag{2.35}$$

from (2.11). There exists the possibility therefore of zero reflection at a particular frequency $\omega = \omega_0$ provided both

$$\kappa_1 = k_0/\rho\omega_0 \quad \text{and} \quad \cot k_0 a = 2k_0 C_1 \tag{2.36}$$

simultaneously. Here $\omega_0^2 = gk_0 \tanh k_0 h$.

In general λ is a complex frequency-dependent parameter and, in terms of λ, we have

$$R_1 \, e^{2ik_0 a} = \frac{\cos k_0 a - \lambda \sin k_0 a}{\cos k_0 a - (\lambda + 2\mathrm{i}) \sin k_0 a} \tag{2.37}$$

and

$$|R_1| = \left| \frac{\cot k_0 a - \lambda}{\cot k_0 a - (\lambda + 2\mathrm{i})} \right| . \tag{2.38}$$

If $|\lambda|$ is small it would appear desirable to choose $a \simeq L/4$, or to position the screen close to a quarter-wavelength $(L/4)$ from the back wall so that $\cot k_0 a = \cot(2\pi a/L)$ is also small. Conversely if $|\lambda|$ is large a spacing of half a wavelength is more appropriate.

In summary, then it would appear that knowledge of C_1, κ_1, and hence λ for the screen enables us to make a sensible choice of where to position the screen relative to the back wall in a given incident wave, so as to minimise $|R_1|$ given by (2.38), and that it is possible, for some value of λ, that $|R_1|$ vanishes for some frequency $\omega = \omega_0$.

As has been mentioned, the problem of more than one screen is complicated and involves products of matrices. There is one case however in which progress can be made. We assume we have n identical screens equally-spaced a distance, say, a apart and having identical permeability coefficients κ_m. Then $\mu = \mu_m$ and $b_m = a$ for all m and (2.24) becomes

$$T_1 = \begin{pmatrix} (1-\mu)\,\mathrm{e}^{\mathrm{i}k_0a} & \mu\,\mathrm{e}^{-\mathrm{i}k_0a} \\ -\mu\,\mathrm{e}^{\mathrm{i}k_0a} & (1-\mu)\,\mathrm{e}^{-\mathrm{i}k_0a} \end{pmatrix}, \tag{2.39}$$

which is independent of m. To obtain an explicit form for $(T_1)^n$ we define, assuming $\mu \neq 1$,

$$\begin{aligned} \cosh A &= \cos k_0a - \mathrm{i}\mu \sin k_0a\,, \\ \cosh B &= -\cos k_0a + \mathrm{i}\mu^{-1} \sin k_0a \end{aligned} \tag{2.40}$$

where A, B are complex. It can be shown that

$$T \equiv T_1^n = \frac{1}{\sinh B} \begin{pmatrix} \sinh(B+nA) & \mathrm{e}^{-\mathrm{i}k_0a} \sinh nA \\ -\mathrm{e}^{\mathrm{i}k_0a} \sinh nA & \sinh(B-nA) \end{pmatrix}. \tag{2.41}$$

For $n=1$ the proof follows, since from (2.40)

$$(1 \mp \mu)\,\mathrm{e}^{\pm\mathrm{i}k_0a} = \cosh A \pm \mu \cosh B\,, \tag{2.42}$$

whence

$$1-\mu^2 = \cosh^2 A - \mu^2 \cosh^2 B \quad \text{or} \quad \mu = \sinh A/\sinh B\,, \tag{2.43}$$

and hence

$$(1 \mp \mu)\,\mathrm{e}^{\pm\mathrm{i}k_0a} = \sinh(B \pm A)/\sinh B\,. \tag{2.44}$$

The proof for general n follows by induction. We notice also from (2.36) the useful result

$$\mathrm{e}^{\pm\mathrm{i}k_0a} = \sinh(B \pm A)/(\sinh B \mp \sinh A)\,. \tag{2.45}$$

It follows from (2.41) and (2.27) with $a_n = na$ that

$$R_n\,\mathrm{e}^{2\mathrm{i}k_0a n} = \frac{\alpha_n + \beta_n}{\gamma_n + \delta_n} = \frac{\sinh(B+nA) + \mathrm{e}^{-\mathrm{i}k_0a} \sinh nA}{\sinh(B-nA) - \mathrm{e}^{+\mathrm{i}k_0a} \sinh nA} \tag{2.46}$$

$$= \frac{\cosh(B+nA) - \cosh(n+1)A}{\cosh(B-nA) - \cosh(n+1)A} \tag{2.47}$$

$$= \frac{\sinh\{\frac{1}{2}(2n+1)A + B\} \sinh \frac{1}{2}(A-B)}{\sinh\{\frac{1}{2}(2n+1)A - B\} \sinh \frac{1}{2}(A+B)}\,. \tag{2.48}$$

after some algebra which makes use of (2.45). Equation (2.46), (2.47) or (2.48) is a general expression for the reflection from n identical equally-spaced screens in terms of the constants A, B defined by (2.40).

For $n=0$ there are no screens and $|R_n| = 1$ as expected. For a single screen, $n=1$ and the expression (2.46) reduces to

$$\frac{\sinh(B+A)+e^{-ik_0a}\sinh A}{\sinh(B-A)-e^{ik_0a}\sinh A}=\frac{\mu(\cosh B+e^{-ik_0a})+\cosh A}{-\mu(\cosh B+e^{ik_0a})+\cosh A}$$

$$=\frac{i\mu(-1+\mu^{-1})\sin k_0a+\cos k_0a-i\mu\sin k_0a}{-\mu(1+\mu^{-1})\sin k_0a+\cos k_0a-i\mu\sin k_0a}\,,\qquad \text{using (2.40)},$$

$$=\frac{\cos k_0a+i(1-2\mu)\sin k_0a}{\cos k_0a-i(1+2\mu)\sin k_0a}\,.$$

in agreement with (2.29).

As a check on the general result we consider $\mu\to 0$ corresponding to continuity of potentials across the screens so that the screens vanish as $\mu\to 0$. We expect $R_n\to 1$.

Now, as $\mu\to 0$,

$$\cosh A=\cos k_0a-i\mu\sin k_0a=\cos(k_0a+i\mu)+O(\mu^2)\,, \tag{2.49}$$

or

$$A\sim i(k_0a+i\mu)\,,\quad \mu\to 0\,, \tag{2.50}$$

whilst

$$\begin{aligned}\mu\cosh B&=i(\sin k_0a+i\mu\cos k_0a)=i\sin(k_0a+i\mu)+O(\mu^2)\,,\\ \mu\sinh B&=\sinh A=i\sin(k_0a+i\mu)+O(\mu^2)\,.\end{aligned} \tag{2.51}$$

Thus

$$R_n\,e^{2ik_0na}\sim\frac{i\sin(k_0a+i\mu)\,e^{nA}-\mu\cosh(n+1)A}{i\sin(k_0a+i\mu)\,e^{-nA}-\mu\cosh(n+1)A}$$

$$\to e^{2ik_0na}\quad\text{as }\mu\to 0\,,\quad\text{or } R_n\to 1\quad\text{as }\mu\to 0\,.$$

For small $|\mu|$ computation of R_n from (2.46) is facilitated by making use of (2.50), (2.51). The condition for R_n to vanish identically is, from (2.48),

$$B=A+2ip\pi\quad\text{or}\quad B+(2n+1)A=2iq\pi\,,\quad p,q\text{ integers}\,, \tag{2.52}$$

provided the denominator of (2.48) does not vanish. The first condition is not admissible since it implies $\sinh B=\sinh A$ or $\mu=1$ from (2.43) which is excluded. The second condition implies

$$\cosh B=\cosh(2n+1)A\,,$$

and it follows from (2.45) that

$$e^{ik_0a} = \frac{\sinh(B+A)}{\sinh B - \sinh A} = \frac{\sinh 2nA}{\sinh(2n+1)A + \sinh A}$$
$$= \frac{\sinh nA}{\sinh(n+1)A} \quad \text{from (2.52)}. \tag{2.53}$$

For example, if $n = 1$ we have the condition

$$e^{ik_0a} = \frac{\sinh A}{\sinh 2A}, \quad \text{or} \quad 2\cosh A = e^{-ik_0a},$$

whence

$$\cosh k_0a = (2\mu - 1)\mathrm{i} \sin k_0a = \lambda \sin k_0a,$$

in agreement with (2.37).

We next consider our second problem. We shall suppose that beyond $x = a_n$ the tank is closed by a vertical rigid wall at $x = a_{n+1} > a_n$. In the absence of the screens the water contained between the walls at $x = 0$ and $x = a_{n+1}$ would oscillate indefinitely with a frequency ω given by $\omega^2 = gk_0 \tanh k_0h$ where $k_0a_{n+1} = m\pi$, $m = 1, 2, \ldots$.

Introduction of the screen will have the effect of damping the motion and producing a complex frequency having negative imaginary part ($e^{-i\omega t}$ assumed). To accommodate all n screens we need only choose

$$A_n e^{ik_0b_n} = B_n e^{-ik_0b_n}$$

to ensure that, from (2.18),

$$\psi'_{n+1}(x) = 0 \quad \text{on } x = a_{n+1}.$$

Then, from (2.25),

$$\begin{bmatrix} 1 \\ e^{2ik_0b_n} \end{bmatrix} A_n = T \begin{bmatrix} 1 \\ 1 \end{bmatrix} A_0,$$

or

$$A_n = (\alpha_n + \beta_n)A_0, \quad A_n e^{2ik_0b_n} = (\gamma_n + \delta_n)A_0,$$

whence

$$(\alpha_n + \beta_n) e^{2ik_0b_n} = \gamma_n + \delta_n \tag{2.54}$$

is the equation for the determination of the k_0.

In the case of a single screen we obtain from (2.24) (2.54), with $m = 1$,

$$\{(1-\mu) e^{ik_0a} + \mu e^{-ik_0a}\} e^{2ik_0b} = (1+\mu) e^{-ik_0a} - \mu e^{ik_0a} \tag{2.55}$$

where

$$\mu_1 \equiv \mu \quad \text{and} \quad b = a_2 - a_1 .$$

Solving for μ gives

$$2\mu \sin k_0 a \sin k_0 b = -\mathrm{i} \sin k_0 c , \quad c = a + b , \tag{2.56}$$

as the equation for the frequencies $\omega^2 = gk_0 \tanh k_0 h$.

A similar equation was obtained in the related problem of a partly immersed vertical baffle in a rectangular wave tank, by Evans and McIver [2] by using a wide-spacing approximation. Here, since the screen extends throughout the fluid the theory is 'exact' under the assumption of the Darcy law (2.12) across the screen.

Now since $\mathrm{i}\mu^{-1} = 2\mathrm{i}\rho\omega c_1/k_0$, $\mu \to \infty$, or $c_1 \to 0$ corresponds to an impermeable screen and equation (2.56) has the solution $\sin k_0 a = \sin k_0 b = 0$ corresponding to undamped free oscillations in the two separate fluid regions. Conversely $\mu \to 0$, $c_1 \to \infty$ corresponds to continuity of potentials across the screen from (2.17) so that in effect the screen disappears. In this case (2.56) has the solution $\sin k_0 c = 0$, being the condition for undamped oscillations in the single larger fluid region of width $c = a + b$.

For finite μ (2.56) provides a complex equation for the determination of the complex wave numbers k_0 and hence the complex frequencies ω from (2.6). If $a = b$, (2.56) reduces to

$$\cos k_0 a = \mathrm{i}\mu \sin k_0 a . \tag{2.57}$$

As for the reflection problem it is possible to make progress in the case of n identical equally-spaced screens by using the explicit expression (2.41) for T. Thus from (2.54), with $b_n = a$, the equation for the complex frequencies becomes

$$(\sinh B)^{-1}\{2 \sinh nA + \mathrm{e}^{\mathrm{i}k_0 a} \sinh(B + nA) - \mathrm{e}^{-\mathrm{i}k_0 a} \sinh(B - nA)\} = 0 ,$$

which, after making use of (2.45), reduces after some algebra to

$$\sin k_0 a \sinh(n+1)A = 0 , \quad \sinh A \neq 0 .$$

The condition $\sinh A \neq 0$ implies $\cosh A \neq \pm 1$ and so rules out the solutions $\sin k_0 a = 0$, and we are left with

$$\sin(n+1)A = 0 \quad \text{or} \quad A = \mathrm{i}p\pi/(n+1) ,$$

whence from (2.40)

$$\cos k_0 a - \mathrm{i}\mu \sin k_0 a = \cos\left(\frac{p\pi}{n+1}\right), \quad p = 1, 2, \ldots, n \quad (\text{but not } p = 0,\ n+1) . \tag{2.58}$$

Note that in the related problem of the partly immersed thin vertical rigid baffle considered by Evans and McIver [2] the solution $\sin k_0 a = 0$ was allowable since a standing-wave solution in each separate region, having zero normal velocity on the boundary of each region is possible. This solution is not available here since from (2.17) this would require ψ to be continuous across, and also to vanish at the boundaries, and there is no such non-trivial solution.

The solution (2.58) for $n = 1$ reduces to (2.57) as required, since $p = 1$ is the only permissible choice. For μ small, (2.58) can be written

$$\cos(k_0 a + \mathrm{i}\mu) = \cos\left(\frac{p\pi}{n+1}\right)$$

so that

$$k_0 a = -\mathrm{i}\mu + \frac{p\pi}{n+1}, \quad p = 1, 2, \ldots, n,$$

and we have n damped modes corresponding to $p = 1, 2, \ldots, n$, all with the same exponential decay.

3. Conclusion

A theory has been presented for determining the effectiveness of a number of thin porous screens in reducing unwanted wave reflections in a narrow wave tank. A general expression (2.29) has been derived for the reflection by a single screen under a Darcy law assumption in terms of assumed known permeability and blockage coefficients given by (2.22) and (2.11). This result was then generalised ((2.47) and (2.40)) to n identical equally-spaced screens by making use of a useful explicit form for the nth power of a 2×2 matrix.

The problem of the wave-damping produced by the introduction of one or n identical screens into a closed rectangular tank was also considered and expressions derived ((2.58) and (2.6)) for the determination of the complex wave frequency ω describing the decay of the waves.

References

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